Why quantum mechanics is weird

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September 20, 2006

Abstract

A trivial artificial universe is used to illustrate why the mathematics of quantum mechanics gives rise to four of its most notorious properties: nondeterminism, interference, disappearance of interference under observation, and entanglement. The artificial universe is designed to be about as simple as it can possibly be while still exhibiting interference; the other three properties naturally emerge from the resulting mathematical framework.

Contents

0 Introduction

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Something similar was done in [Dr88]. However, the artificial universe in that paper is more complicated than the one used here, perhaps because its objectives were less narrowly focused.

1 Classical physics

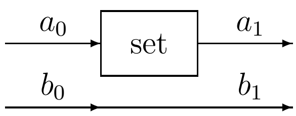
We begin with a "classical" artificial universe.

At any given moment, the state of our classical universe consists of two boolean (i.e., true/false) variables, which we will call a and b. The universe has just four possible classical states.

We are also allowed to set up some experimental apparatus that causes changes in the state of the universe over time; for this, we have three kinds of devices:

- A *set* device, which can be applied to either variable, causing its value to become *true*.
- A *clear* device, which can be applied to either variable, causing its value to become *false*.
- A *copy* device, which assigns the value of one variable to the other variable.

We will use diagrams to depict our experimental setups. The variables are shown as left-to-right arrows, and the devices are shown as boxes.

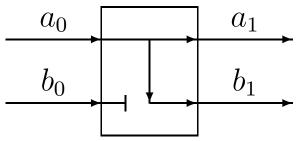


Experiment 1.

Notation x_k indicates the value of variable x at time k. Here, a becomes true and b doesn't change:

$$\begin{aligned} a_1 &= \text{true} \\ b_1 &= b_0\,. \end{aligned} \tag{1}$$

A copy device is shown as a box straddling both variables, with the internal data paths shown, like this:



Experiment 2.

The behavior of this system is

$$\begin{aligned} a_1 &= a_0 \\ b_1 &= a_0\,. \end{aligned} \tag{2}$$

2 Quantum physics

Now we define a "quantum" version of our artificial universe.

At any given moment, the state of our quantum universe is a weighted set of the four classical states (called a superposition). The weight on each classical state is a real number.[1] Only the relative weights are significant in identifying the superposition: if all the weights are multiplied uniformly by some non-zero constant c, the resulting weights are an equivalent representation of the same superposition.

We associate with each superposition a probability distribution of the classical states, in which the probabilities of the classical states are proportional to the squares of their weights. The actual probability of each classical state is therefore the square of its weight divided by the sum of the squares of all the weights in the superposition (because the sum of the probabilities must be equal to one). The *normal* representation of the superposition is the representation in which the sum of squares of weights is equal to one, so that the probability of each classical state is just the square of its weight. Normalization uniquely represents each superposition (up to a sign change, i.e., multiplying all weights by -1) as a real 4-vector of unit length, and the set of all possible superpositions is isomorphic to one hemisphere of the set of all real unit 4-vectors.

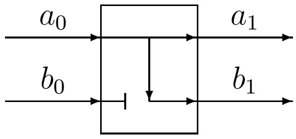
Suppose a quantum state q is acted on by an experimental device d.[2] Each classical state $c \in q$ has one or more successor classical states in q', to which it contributes some weight (possibly zero). We require that the sum of the squares of these contributions is equal to the square of the weight of c in q. A classical state $c' \in q'$ may

[1]We could have made the weights complex, or even (if we wanted to be especially realistic) quaternion; but we're trying to keep our model very simple, and for interference it's enough that the weights can have different signs, so that they cancel each other. We considered and rejected the idea of restricting the weights even more, to integer values, or even to the finite set $\{-1, 0, 1\}$, because using the real continuum allows us to maintain the familiar relation of quantum mechanics between weights and probabilities.

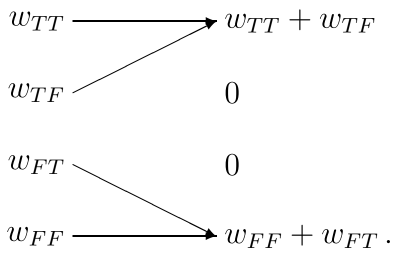
[2]It would be straightforward to deal with multiple devices in parallel, but since there are only four ways that can happen in our universe, and we won't be using any of them, there's no reason to put ourselves through the tedium.

get contributions from several predecessors in q. The contributions to each $c' \in q'$ are summed, giving an unnormalized weight for c' in q'; if we want to recover the probabilities of different classical states in q', we then renormalize the weights in q' by dividing them by the sum of their squares. We'll work through some examples in detail, momentarily.

For each quantum experiment we consider, beneath our diagram depicting the experimental setup (as in the classical case, §1) we will provide a second diagram using arrows to depict the flow of weights from initial to final quantum state. The four weights of each quantum state are shown in a column directly below the corresponding variables in the experimental setup, in the order TT (that is, $\langle a, b\rangle = \langle \text{true}, \text{true}\rangle$), TF ($\langle a, b\rangle = \langle \text{true}, \text{false}\rangle$), FT, FF; the initial weights are called w_{TT}, w_{TF}, w_{FT}, w_{FF}. In the case of the *copy a* device (Experiment 2),



Experiment 2q



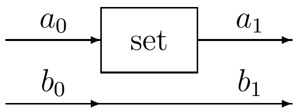
Here are all the basic rules for how a classical state $c \in q$ with weight $w(c)$ contributes weights $w'(c{\to}c')$ to classical states $c' \in q'$, when acted on by device d.

- Case I: d is a copy device (as above, Experiment 2q). The effect is just as in classical physics. Let c' be the classical state that results in classical physics from applying d to c; then $w'(c{\to}c') = w(c)$.

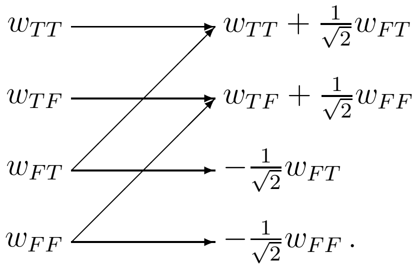
- Case II: d is a *set* device on variable a. If a is true in c, nothing changes; that is, $w'(c{\to}c) = w(c)$. If a is false in c, let c' be the classical state that differs from c only by making a true; then the weight is split equally between c and c', but with the sign of the weight on c reversed; that is, $w'(c{\to}c') = \frac{w(c)}{\sqrt{2}}$ and $w'(c{\to}c) = -\frac{w(c)}{\sqrt{2}}$.

- Other cases: A *set* device on variable b works the same way. A *clear* device works symmetrically to a *set*: it leaves a false variable alone; and if applied to a true variable, it gives the two outcomes equal weights but negates the weight of the no-change outcome.

For example, here is the quantum version of *set a* Experiment 1:



Experiment 1q



By a sufficiently close scrutiny of this weight-flow diagram, we could find in it the mathematical signatures of all four weird quantum behaviors to be studied later in the paper — *if* we already knew what to look for. To compellingly demonstrate what the mathematical signatures are, each behavior will be exhibited in §3 in an experimental setup tailored to that behavior.

One feature of this experiment, prominent in the diagram, that is *not* necessary to §3 is the non-classical proportions between probabilities caused by the squares-of-weights probability distribution.

For any pure initial state, i.e., an initial quantum state in which one classical state has probability 1 and the rest have probability 0, the final probabilities for this experiment are just what one would expect classically (stipulating nondeterministic outcome for *set*ting a false value, with a 50/50 distribution). However, in the case of uniform initial weights, $\langle \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \rangle$ with probability distribution $\langle \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \rangle$ (ordered as in the diagram: TT, TF, FT, FF), the final weights will not produce the classically expected probability distribution: the classical expectation would be probabilities $\langle \frac{3}{8}, \frac{3}{8}, \frac{1}{8}, \frac{1}{8} \rangle$, while here we get probabilities $\langle \frac{3+2\sqrt{2}}{8+4\sqrt{2}}, \frac{3+2\sqrt{2}}{8+4\sqrt{2}}, \frac{1}{8+4\sqrt{2}}, \frac{1}{8+4\sqrt{2}} \rangle$.

By choosing a different constant of proportionality in the rule for *set a* ($\frac{1+\sqrt{3}}{2}$ rather than $\frac{\sqrt{2}}{2}$), we could make both the $a_0 =$ false and the uniform-weight cases work out classically (in the latter case, initial weights $\langle \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \rangle$ would produce final weights $\langle \frac{3+\sqrt{3}}{4}, \frac{3+\sqrt{3}}{4}, -\frac{1+\sqrt{3}}{4}, -\frac{1+\sqrt{3}}{4} \rangle$, with probability distribution $\langle \frac{3}{8}, \frac{3}{8}, \frac{1}{8}, \frac{1}{8} \rangle$). However, as long as we take probability to be the sum of squares of weights, and contributions from each initial weight are determined independently and then added, there is no way to retain the classical rules of probability combination in every case, even for this simple experiment, and even if we consider only nonnegative initial weights.

The squares-of-weights probability distributions are characteristic of quantum mechanics, and as such we expect that they may provide deeper insights into the quantum mathematical arrangement; nevertheless, in pursuit of our immediate goal we *would* abandon the squares-of-weights if it appeared to be complicating the demonstrations of quantum effects. As squaring the weights is a convenient way of extracting nonnegative probabilities from the necessarily signed weights (the signs are essential to interference and entanglement), we have chosen to maintain the squaring. In doing so, however, we note —and the reader may verify, as situations arise later in the paper— that the basic demonstrations of non-classical effects in §3 would still work if we had used $p_c = \mathrm{abs}(w_c)$ (with corresponding changes to the propagation rules) rather than $p_c = w_c^2$.

3 Non-classical effects

3.1 Nondeterminism

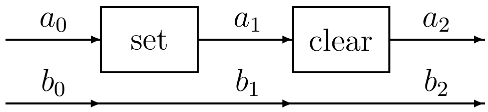
Our artifical quantum universe "obviously" has the property of nondeterminism, because a given classical state can have multiple successor classical states. But for a given *quantum* state, there is only one successor quantum state; so in that sense our quantum physics is strictly deterministic.[3] Quantum nondeterminism could therefore —again, *in a sense*— be considered to be merely an illusion that is produced when we try to view what is happening as a sequence of classical states. This is, more or less, the so-called "many-worlds" interpretation of quantum mechanics.

3.2 Interference

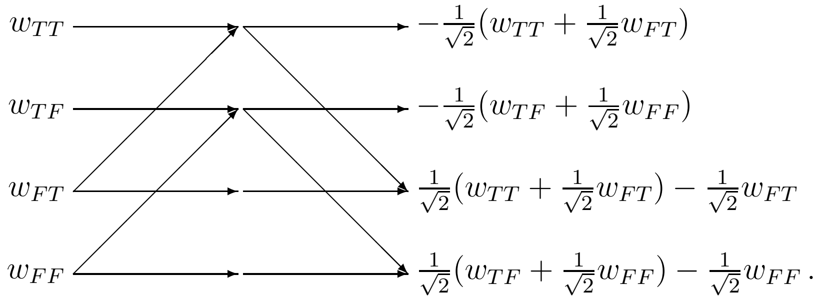
Our objective criterion for interference will be that an experiment involving two devices, acting on a pure initial state, produces a final probability distribution that doesn't obey the classical rules for merging the final probability distributions for the individual devices.

Consider the following experimental apparatus.

[3]In choosing the rules for our quantum universe, we haven't gone to any trouble to guarantee that interference won't send all the weights to zero simultaneously. If that happened, we'd find ourselves with a "successor quantum state" that can't be normalized because the vector has length zero. In effect, the preceding quantum state would have *no* successor, rather than exactly one successor; but this anomaly, even if accidentally possible under the rules we've chosen, would have no bearing on the purpose of the paper.



Experiment 3



Device *set a* alone, given a pure input state with a_0 = true, would produce the same pure state as output; while, given a pure input state with a_0 = false, it would produce output with equal probabilities of a_1 being true or false. Symmetrically, *clear a* alone, given a pure input state with a_0 = false, would produce the same pure state as output; while, given pure input state with a_0 = true, it would produce output with equal probabilities of a_1 being true or false. By classical rules for combining probabilities, we would expect Experiment 3 on a pure input state with a_0 = false to produce output with a 25% probability of a_1 = true.

Putting the four pure initial states through the entire mathematical transformation of Experiment 3,

initial weights	final weights	probabilities
$\langle 1,0,0,0\rangle$	$\langle \frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0\rangle$	$\langle \frac{1}{2}, 0, \frac{1}{2}, 0\rangle$
$\langle 0,1,0,0\rangle$	$\langle 0, \frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\rangle$	$\langle 0, \frac{1}{2}, 0, \frac{1}{2}\rangle$
$\langle 0,0,1,0\rangle$	$\langle \frac{-1}{\sqrt{2}}, 0, \frac{1-\sqrt{2}}{2}, 0\rangle$	$\langle \frac{10+4\sqrt{2}}{17}, 0, \frac{7-4\sqrt{2}}{17}, 0\rangle$
$\langle 0,0,0,1\rangle$	$\langle 0, \frac{-1}{\sqrt{2}}, 0, \frac{1-\sqrt{2}}{2}\rangle$	$\langle 0, \frac{10+4\sqrt{2}}{17}, 0, \frac{7-4\sqrt{2}}{17}\rangle$.

(3)

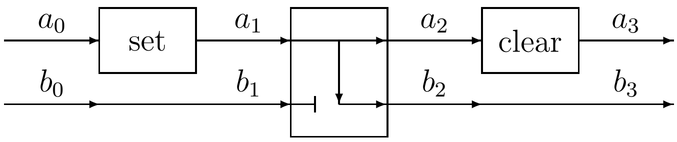
The first two of these results are what we would expect from classical probability: the *set a* device doesn't disturb a_0 = true, and there is a 50% chance that the *clear a* device will change a. For the a_0 = false initial states, though, the probability of a = false in the final state (a_2) is only about 8%, rather than the classically predicted 25%.

If we had abandoned the squares-of-weights rules for probabilities, and taken instead $p_c = \text{abs}(w_c)$, the probability of a_2 = false given a_0 = false would have been *zero*.

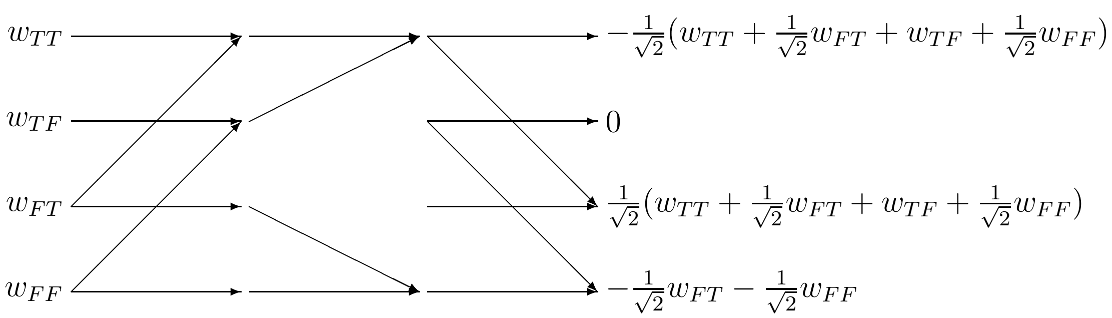
Evidently, interference can arise in our quantum universe when a single successor classical state has multiple predecessor classical states.

3.3 Observation

Now, let's modify our apparatus by "observing" the value of a in the middle of the experiment. We do this by *copy*ing a to b after the *set* and before the *clear*.



Experiment 4



Putting the four pure initial states through the experiment,

initial weights	final weights	probabilities
$\langle 1, 0, 0, 0\rangle$	$\langle \frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0\rangle$	$\langle \frac{1}{2}, 0, \frac{1}{2}, 0\rangle$
$\langle 0, 1, 0, 0\rangle$	$\langle \frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0\rangle$	$\langle \frac{1}{2}, 0, \frac{1}{2}, 0\rangle$
$\langle 0, 0, 1, 0\rangle$	$\langle \frac{-1}{2}, 0, \frac{1}{2}, \frac{-1}{\sqrt{2}}\rangle$	$\langle \frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}\rangle$
$\langle 0, 0, 0, 1\rangle$	$\langle \frac{-1}{2}, 0, \frac{1}{2}, \frac{-1}{\sqrt{2}}\rangle$	$\langle \frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}\rangle$.

(4)

The probabilities are exactly as predicted by classical rules.

Mathematically, interference can only happen when the *entire classical state of the universe* is identical for two or more contributions to the new superposition. Thus, the interference must vanish in the presence of observation, *even if the observation doesn't affect the thing observed*, because observation means by definition that the classical states of the universe are no longer identical.

3.4 Entanglement

Our objective criterion for entanglement will be that an experiment not involving b alters the probability distribution for b. Precisely, we wish to devise a sequence of two experiments, call them S and T, such that T does not involve b, but the probability of $b =$ true after just S is different than the probability after the sequence S, T.

The only experimental operations that don't involve b are *set* a and *clear* a, so T has to be built up out of these. We take T to be the set-clear sequence, Experiment 3, which exhibits quantum interference.

From the mathematical transformation performed on the initial quantum state by Experiment 3, it is immediately apparent that

(1) given any pure initial state, $b_2 = b_0$.

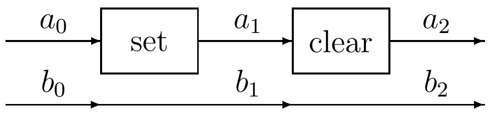
(2) given the uniform initial state, $\langle \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \rangle$, the probability that $b_2 =$ true is $P(b_2) = 0.5$.

On closer inspection,

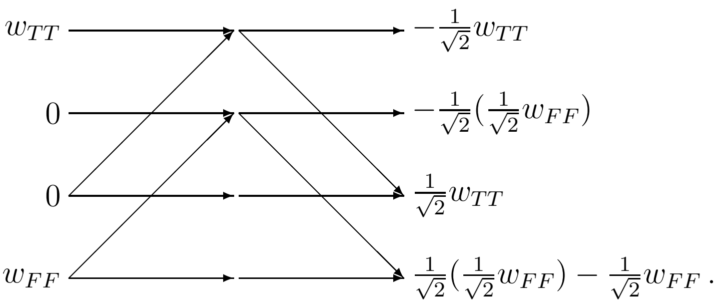
(3) if $P(b_0) = 1$ then $P(b_2) = 1$; if $P(b_1) = 0$ then $P(b_2) = 0$.

(4) if the ratio of w_{TT} to w_{FT} equals the ratio of w_{TF} to w_{FF} (that is, $w_{TT}w_{FF} = w_{TF}w_{FT}$), then $P(b_0) = P(b_2)$.

With these properties of T in mind, we take S to be a copy device (it doesn't really matter which one). Then for the initial state of T, $w_{TF} = w_{FT} = 0$. As long as $w_{TT} \neq 0$ and $w_{FF} \neq 0$, $w_{TT}w_{FF} \neq w_{TF}w_{FT}$. Plugging $w_{TF} = w_{FT} = 0$ into T,



Experiment 5



The unnormalized final weights are $\langle \frac{-1}{\sqrt{2}}w_{TT}, \frac{1}{2}w_{FF}, \frac{1}{\sqrt{2}}w_{TT}, \frac{1-\sqrt{2}}{2}w_{FF} \rangle$; the sum of squares of weights is $\frac{1}{2}w_{TT}^2 + \frac{1}{4}w_{FF}^2 + \frac{1}{2}w_{TT}^2 + \frac{3-2\sqrt{2}}{4}w_{FF}^2 = w_{TT}^2 + \frac{2-\sqrt{2}}{2}w_{FF}^2$. So,

$$
\begin{aligned}
P(b_0) &= \frac{w_{TT}^2}{w_{TT}^2 + w_{FF}^2} \\
P(b_2) &= \frac{w_{TT}^2}{w_{TT}^2 + \frac{2-\sqrt{2}}{2}w_{FF}^2}
\end{aligned}
\tag{5}
$$

and, assuming $w_{TT} \neq 0$ and $w_{FF} \neq 0$, $P(b_2) > P(b_0)$.

Mathematically, entanglement occurs here because the set/clear experiment (T) manipulates a in ways that cause interference only when b is false; so even though T doesn't affect b in the classical sense, the act of renormalizing the final weights skews the probablity of $b =$ true.

4 Analysis

Nondeterminism occurs, or perhaps *appears*, in a quantum universe, when a classical state of the universe has multiple successors. Interference occurs when a classical state of the universe has multiple predecessors. Interference disappears when an observation distinguishes between predecessor states that would otherwise interfere with each other, exactly because "distinguishing" means, by definition, that the successor states of the universe are no longer identical. Entanglement is the most subtle effect: it occurs when interaction with one facet of classical state causes interference that is asymmetric with respect to a different facet of classical state.

All four effects follow from the quantum view of the universe as consisting of weighted superpositions of classical states. This conclusion should be understood as narrowly as possible; it explains why quantum-style mathematical models exhibit these four effects, and *nothing else*. In particular, we cannot draw any negative conclusions about the quantum mechanics of the real world by studying this artificial universe. Our model is designed to have general properties that hold for all quantum-style mathematical models; but we don't expect the real world to have only general properties, rather we expect the real world to have very special properties.

With that in mind, here are two basic questions that our study puts in greater relief. Quantum mechanics portrays physics as a deterministic succession of superpositions of weighted classical states; so,

1. What, intrinsically, *are* classical states, that they should play such an essential role in the mathematics? and,

2. What, intrinsically, are the weights in a superposition of classical states, that their ability to cancel each other should be so fundamental to the non-classical character of quantum reality? (Keep in mind that in the quantum mechanics of the real world, these weights have quaternion structure.)

A Document history

This document is the main instrument of my efforts to understand the conceptual peculiarities of the mathematics of quantum mechanics (note: peculiarities of the math, *not* peculiarities of quantum mechanics itself which, as Richard Feynman famously observed, nobody understands). I started it in 2002, to sharpen ideas of [Dr88]. A long-standing interest in the hypothesis of meta-time eventually led me to add entanglement to the set of properties explored by the document in June–July 2005, adding weight-flow diagrams at the same time.

November 2005: Improved the analysis section, and put the document on the web.

September 2006: Clarified the discussion of weight squaring at the end of §2.

References

[Dr88] Gary L. Drescher, *Demystifying Quantum Mechanics: A Simple Universe with Quantum Uncertainty*, memo 1026a, MIT AI Lab, revised December 1988. Available (as of November 2005) at URL: `http://www.ai.mit.edu/research/publications/`